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Dark Energy versus an Inhomogeneous Universe

Abstract

Scientists were surprised to discover that the universe is apparently accelerating. They expected the effect of gravity would be to cause a deceleration of the expanding universe. After interpreting astronomical observations in terms of acceleration, they began to develop new structural models of the universe. At present, the prevailing model involves the existence of dark energy as described by the cosmological constant in a homogeneous and isotropic universe (a universe containing the same properties and structure everywhere). Dark energy is the term used to describe the idea of a repulsive force which counteracts gravity's attractive force and leads to acceleration. An opposing view holds that the universe is inhomogeneous and isotropic. Interaction of the component parts of the universe could account for observations in the absence of dark energy. The former is called the Friedmann-Lemaître-Roberston-Walker universe model and the latter is the Lemaître-Tolman-Bondi model. Theoretically both models provide solutions to

Einstein's gravity equations in general relativity. Observationally both models explain cosmic microwave background radiation (CMB) and supernovae type one "a" (SNeIa). Scientists need more observational information in order to determine which model best represents the universe.

I. Introduction

The two conflicting structural models of the universe, one that assumes dark energy versus one based on an inhomogeneous mass distribution, are rich with historical background. For this reason, I will present my paper in a chronological fashion. Beginning with Newton and his law of gravity, I will continue on to Einstein and general relativity. Through the modeling of Friedmann and the observations of Hubble in the 1920's, scientists concluded the universe was expanding. With Robertson and Walker's exact solutions to the Einstein field equations, the Friedmann-Lemaître-Roberston-Walker universe model was established. Scientists introduced dark energy (year 2000) to explain observations of the cosmic microwave background radiation (CMB) discovered in 1964 and distant supernovae studied in 1999. Also in 2000, Célérier published her article about the Lemaître-Tolman-Bondi model of an inhomogeneous universe in order to explain the CMB and supernovae observations. Also presented in this paper are results published by many researchers who comment on the advantages and disadvantages/shortcomings of each model.

II. Isaac Newton and Albert Einstein

This section discusses the development of the concept of gravity and its effects on universe models. Gravity is an important factor in models of the universe. First discussed is Newton who formulated an equation to describe his observations of gravity. Second Einstein is discussed. He redefined gravity and introduced curved spacetime and the cosmological constant Λ. Today Λ is popularly used to characterize dark energy.

In 1687, Isaac Newton published *Philosophiae Naturalis Principia Mathematica*. He observed that there is an attraction between any two masses. In this book, Newton described the relationship as the universal attraction of gravity

$$F_{grav} = \frac{GM_1 M_2}{r^2}. \qquad (1)$$

This equation became known as the universal law of gravitational attraction. Imagine a space with two bodies. One body with mass M_1 is separated by a distance r from the other mass M_2. Newton described gravity as the attractive force F_{grav} that M_1 exerts on M_2 and *vice versa*. The gravitational constant G is a constant of proportionality that is equal to 6.6742 x 10^{11} N m^2 kg $^{-2}$.

Newton described the attraction of gravity between two bodies which are in a Euclidean space geometry. This type of geometry involves three-dimensional spatial coordinates as functions of time $(x(t), y(t), z(t))$ [1, 2]. Euclidean geometry has straight line geodesics, which are the paths of free particles representing the shortest distance between any two points along them [1].

Distances in Euclidean space can be described by the distance vector **l**. A metric equation is a distance function which defines a distance between points described by position components x, y, z. The metric equation that represents **l** is

$$l^2 = (x^1)^2 + (x^2)^2 + (x^3)^2 = \sum_{i,j=1}^{3} g_{ij} x^i x^j = g_{ij} x^i x^j . \tag{2}$$

The components x^1, x^2, x^3 have replaced x, y, z [1]. The quantities g_{ij} are the nine components of the metric tensor **g** [1]. In general, a tensor is a mathematical generalization of the idea of a vector to a symbol that actually represents several other vectors at once [2]. Each component or piece of the tensor is a vector [2]. In the equation above, **g** is a tensor and g_{ij} is a vector component or piece of **g**. The i and j are indices or labels attached to components of vectors or tensors for the purpose of indicating to which directions the components are associated [2].

Tensors can be defined to have one or more indices [2]. The indices i and j above represent the numbers 1, 2, 3. For example, the different components can be written as g_{11}, g_{21}, g_{31}, and so on. The metric tensor **g** is specifically a mathematical structure, represented by a matrix with components having two indices, which contains all the information about the intrinsic geometry of the space it describes [1,2]. Here in Euclidean three-dimensional space the components of **g** are equal to the Kronecker delta δ,

$$g_{ij} = \delta_{ij} = 1 \text{ if } i = j \text{ or } 0 \text{ if } i \neq j. \tag{3}$$

Over 200 years after Newton, Albert Einstein began to study gravity and the geometry of space. He disagreed with Newton's ideas on space and gravity. Thus, Einstein developed the theory of special relativity and general relativity to redescribe space and gravity. He used special relativity and spacetime distance to build up to his general theory of relativity.

Special relativity describes different inertial reference frames that are in relative motion at a constant velocity. An object in an inertial frame remains at rest or in uniform motion unless a force is acted upon it. Once acted upon the object accelerates. Einstein wanted to know how the frames exchange signals or measure objects between themselves [1]. He postulated two ideas in special relativity [1]. The first says that when measurements are made in one inertial frame, the same results will be obtained if the same phenomenon is measured in a different frame [1]. This type of phenomenon is called invariant [1]. Variant phenomena have different numerical measurement results when studied from different inertial frames [1]. The second idea of special relativity is that light travels at a constant speed c in vacuum in all frames [1].

The theory of special relativity can be expressed in terms of the Lorentz transformations where time and space are variants and their measurements change from one reference frame to another [1]. In special relativity the Lorentz transformation equations are used to change the four-dimensional spacetime coordinate system (time t, x, y, z) of one observer to another observer [2]. Spacetime is the set of all points having a fixed location and time of occurrence [2].

The equation for the spacetime distance between two nearby events is

$$ds^2 = c^2d\tau^2 = c^2dt^2 - dx^2 = c^2dt'^2 - dx'^2. \quad (4)$$

Here x and x' are two linear axes in one-dimensional space [1]. Axis x' is at rest and x is moving with constant velocity v in the positive x' direction [1]. Time intervals are represented in the two coordinates as dt and dt' [1]. Scalar products are invariants under Lorentz transformations, but as stated earlier, time intervals dt, dt' and space coordinates x, x' are variants [1]. The invariant replacements for x and t are proper distance ds and proper time $d\tau$ [1].

After developing special relativity, Einstein wanted to develop a new theory of gravity. He had several reasons for why space and gravity had to be reconceptualized. He realized that the geometry of space was not flat but was curved. The mass-energy and momentum content of matter caused spacetime to curve. Also, Newton's law of gravitation (1) conflicted with the special theory of relativity, and relativity had limitations as well [1]. Einstein's theory requires gravity fields and matter to be in inertial reference frames and does not consider other types of frames, but Einstein wanted the new gravity theory to use equations that described the fields as being the same in any coordinate system [1, 2].

At last, Einstein was able to formulate the gravitational equation for general relativity:

$$G_{\mu\upsilon} = \frac{8\pi G}{c^4} T_{\mu\upsilon}. \quad (5)$$

Here μ and ν are used because the tensors $G_{\mu\nu}$ and $T_{\mu\nu}$ are rank two; they have ten independent components in four dimensions [2]. In other words, $G_{\mu\nu}$ and $T_{\mu\nu}$ each represent sixteen different equations. These differ from the metric tensor **g** mentioned above, since **g** has three dimensions and the indices used are i, j. The Greek μ, ν are spacetime indices and run over 0, 1, 2, 3; therefore, $x^0 = t$, $x^1 = x$, $x^2 = y$, $x^3 = z$. $T_{\mu\nu}$ is called the stress-energy tensor [1, 2]. This tensor represents the source of gravity in general relativity [2]. The sources of gravity included in $T_{\mu\nu}$ are the mass-density, momentum-density, and pressure [2]. The component tensors are stress-energy tensors as well. They represent sources of energy such as baryons (a collective name for protons, neutrons, and related unstable particles of larger mass [2]), radiation, and other possible forms [1]. $G_{\mu\nu}$ is called the Einstein curvature tensor [1, 2]. As opposed to the flat spacetime of Euclidean geometry, Einstein uses $G_{\mu\nu}$ to describe the curvature of all points of space and time. "The Einstein tensor is the mathematical construction that he used to describe the part of the curvature of spacetime that directly equals the densities, momenta, and stresses of the matter producing gravity" [2]. $G_{\mu\nu}$ contains only components which are either quadratic in the first derivatives of the metric tensor $\mathbf{g}_{\mu\nu}$ or linear in the second derivatives [1]. Einstein's gravity equation in general relativity expresses that the mass-density, momentum-density, and pressure embodied in the stress-energy tensor determines the geometry of spacetime [1]. This in turn determines the motion of matter [1].

Einstein's theory of general relativity predicted a dynamical universe, and Einstein assumed this was erroneous [2]. The leading thought early in the twentieth century was that the universe was and always had been static (neither expanding nor contracting) [2]. Einstein did not want to abandon the theory because it successfully predicted how gravity behaves in the solar system [2]. Einstein began to formulate a correcting factor which would make his theory of general relativity predict a static universe [2]. In 1917, Einstein introduced into his equation for gravity the term $\Lambda g_{\mu\upsilon}$, an invariant constant that did not change values from one reference frame to another [1, 2]

$$G_{\mu\upsilon} + \Lambda g_{\mu\upsilon} = \frac{8\pi G}{c^4} T_{\mu\upsilon} \cdot \tag{6}$$

He defined Λ as the cosmological constant and gave it specific characteristics which are constant in time and in space and are the same no matter what observer measures them [2]. Note that the following description of Λ is popularly used to describe dark energy as well. In this paper, "dark energy" refers to the cosmological constant and vice versa. The main characteristics of the cosmological constant are positive density and negative pressure. Λ can be viewed as a physical fluid, call it the "cosmological fluid" in which pressure p_Λ and density ρ_Λ are related by the equations

$$p_\Lambda = -\rho_\Lambda c^2 \tag{7}$$

or

$$p_\Lambda = -\frac{\Lambda c^2}{8\pi G}. \tag{8}$$

Einstein introduced the cosmological fluid in order for general relativity to allow for a static universe as opposed to a dynamic one [2]. Matter has the attractive force of gravity and this force was causing Einstein's gravity equation to be dynamic [2].

Including the cosmological constant, Einstein was able to successfully develop a new theory of gravity (general relativity) which is able to explain observations of gravitational effects. He brilliantly formed the mathematical equations for gravity (6). But Einstein did not notice that the adaptations he made to account for a static Universe caused his equations for gravity to be unstable [1]. It was not until 1930 that Arthur Eddington noticed the flaw [1]. An imbalance between Λ and ρ would cause the Universe to either accelerate into expansion or decelerate into contraction [1]. In 1929, one year before Eddington's discovery about the gravity equation, Edwin Hubble discovered that the Universe was expanding; and Einstein decided to drop the static universe theory as well as the cosmological constant [1].

III. Friedmann-Lemaître-Robertson-Walker (FLRW) Universe Model

In this section I will discuss the development of the Friedmann-Lemaître-Robertson-Walker (FLRW) model. It describes a homogenous, isotropic, and expanding/contracting universe. In its beginning it successfully described the observations of the universe without Einstein's cosmological constant. The FLRW model has needed to accept dark energy (in this paper referring to the cosmological constant) in order to explain recent observations of supernovae. The adaptations of the FLRW model will be discussed in section V. Today this is the most often used structural model of the universe.

In 1922, Alexander Friedmann solved Einstein's gravity equations (5) (without the cosmological constant) for a universe that is homogeneous, isotropic, and either expanding or contracting. The solutions are

$$\frac{\dot{R}^2 + kc^2}{R^2} = \frac{8\pi G}{3}\rho\ , \tag{9}$$

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2 + kc^2}{R^2} = -\frac{8\pi G}{c^2}p\ . \tag{10}$$

Here R is a cosmic scale factor and is dependent on time [1]. k is the curvature parameter [1]. If k is taken as +1, 0, or -1 then the corresponding curvature of space is the three-sphere, flat three-space, or the three-hyperboloid. Friedmann's solution will be discussed further in the paper. Friedmann described the universe according to the Cosmological Principle. This principle states that the universe is homogeneous and isotropic in three-dimensional space, has always been so, and will always remain so [1].

Homogeneity is a property of a distribution that is uniform [1]. It is the concept of a geometrical space in which every point is as good as any other point at any given instant of time [1, 3]. For example, smooth peanut butter is homogeneous. No matter what point one is at in the peanut butter, the observer will experience the same smooth texture. The homogeneity of the universe cannot be determined by observations in just one region such as the earth. Observers on earth cannot determine whether all of space at a given time has the same geometry as the space near Earth [1].

Isotropy [2] is a characteristic of space that indicates it has the same properties in all spatial directions [1, 3]. Isotropy deals with a specific spot in space. No matter what direction from this point he/she looks, the observer will see the same properties in space [1]. With earth as the chosen point, an observer can determine if the space surrounding the earth is isotropic [1]. For an example, a bird in the center of a tree experiences isotropy. The bird is surrounded by the branches, then by the leaves, then by the flowers on the tree, then by the sky. From every direction the bird looks, it sees the same properties and patterns surrounding it.

It turns out that a Newtonian version of Friedmann's solution to Einstein's equations can be derived in the absence of pressure (*i.e.*, for dust) by Newton's second law of motion and his universal gravitation law. Friedmann's homogeneous and isotropic parameters simplify the difficult process of solving Einstein's equations. In the Newtonian version, Friedmann's universe can be represented as a large gas cloud [4]. Imagine a particle which is at a radius vector **r** from a comoving observer (one who views an isotropic universe). The radius vector **r** is related the velocity **v** of a particle by the equation

$$\mathbf{v} = f(t)\,\mathbf{r}, \tag{11}$$

where $f(t)$ is a function of time t [4]. This velocity-distance relation can be called the velocity law. By integrating the velocity law, one can find the position of the particle

$$\mathbf{r} = R(t)\,\mathbf{r}_0. \tag{12}$$

Differentiate equation (12) and compare it to the velocity law. One can see that $R(t)$ is related to $f(t)$ by the equation

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \dot{R}\mathbf{r}_0 = f(t)\mathbf{r}\,. \tag{13}$$

In (12) $\mathbf{r}_0$ is the position of the particle at time t_0, therefore

$$R(t_0) = 1. \tag{14}$$

The position of a particle in the gas cloud is dependent on the time-dependent scale factor $R(t)$, as seen from (12). Since all the particles are homogenous and isotropic, their collective motion determines the motion of the gas cloud. The cloud either uniformly expands or contracts. Just like the position of particles that compose it, the cloud's motion is dependent on the scaling factor $R(t)$. By plugging $f(t)$ from (13) into (11), we obtain

$$\mathbf{v} = \frac{\dot{R}}{R}\mathbf{r}\,. \tag{15}$$

The way velocity and position relate to each other is dependent on the function of time $R(t)$. The following equation for $R(t)$ is the solution to Einstein's gravity equation. Here $R(t)$ is derived by using Newton's second law of motion and his law of gravitation

$$\ddot{\mathbf{r}} = -\frac{GM}{r^3}\mathbf{r}\,,$$

$$M = \frac{4\pi}{3}\rho(t)r^3 = \frac{4\pi}{3}\rho(t_0)r_0^3\,,$$

$$\rho(t) = \frac{\rho(t_0)}{R^3}\,,$$

$$R^2\ddot{R}+\frac{4\pi}{3}G\rho(t_0)=0\,. \qquad (16)$$

G is the gravitational constant, M and $\rho(t)$ are the mass and density of the cloud, t_0 is standard time. The cloud will be static when the rate of change its radius with respect to time and its acceleration is zero

$$\dot{R}=\ddot{R}=0\,. \qquad (17)$$

From the previous equations, (17) will not occur unless the density of the cloud ρ is equal to zero. In order to solve for $\dot{R}$, multiply $\ddot{R}$ equation in (16) by $\dot{R}/R^2$ and integrate with respect to time t to get

$$\dot{R}^2=\frac{8\pi}{3}\frac{G\rho(t_0)}{R}-k\,, \qquad (18)$$

where k is a constant of integration. For a homogeneous and isotropic universe model, there are three choices for the value of k,

$$k=0$$

$$k>0$$

$$k<0. \qquad (19)$$

In general relativity, k gives the curvature of three-dimensional space at any time t_0 [4]. When $k = 0$, the model universe is often called flat. It is the flat three-dimensional Euclidean space. Choose a point in this space (any point since the model is homogeneous and isotropic). Form a surface by drawing lines of equal length (radii) from the chosen point. Draw a circle connecting all the radii of equal length. The length of this circle is its circumference. In flat Euclidean space, the ratio of the circumference to the radius is equal to 2π.

When $k > 0$, the universe model is commonly called closed and the space is called spherical. As above, choose a point in the space and draw radii of equal length. Connect the radii by drawing a circle around them. The ratio of the circumference to the radius is less than 2π. This is because the space is not flat as in Euclidean space. It is positively curved. The third universe model when $k < 0$ describes a space in which the circumference of a circle to the radius is greater than 2π. This model is called open or hyperbolic.

At first Einstein rejected Friedmann's solutions to his equations. He considered the universe was static, whereas the solutions call for a universe either expanding or contracting. As mentioned above, the cloud would need density ρ equal to zero to be static. Einstein's mindset was changed in 1929 when Edwin Hubble discovered observational evidence that the universe was expanding. He established that the recessional velocity of a galaxy is proportional to its distance.

From 1922 to 1924 Georges Lemaître solved Einstein's gravitational equations (without knowledge of Friedmann's work) and discovered the same solutions as Friedmann. In 1934 Howard Percy Robertson and Arthur Geoffrey Walker showed that a homogeneous and isotropic universe model had no other solutions to Einstein's equations except for Friedmann and Lemaître's work. Thus with the work of Georges Lemaître, Howard Percy Robertson, and Arthur Geoffrey Walker, the FLRW universe model was developed. This model describes a homogenous, isotropic, and expanding or contracting universe.

IV. Cosmic Microwave Background (CMB)

Both models of the universe FLRW and the Lemaître-Tolman-Bondi model call for isotropy. An important observational support for isotropy is the cosmic microwave background radiation (CMB). This section discusses the discovery and importance of CMB.

Arno Penzias and Robert Wilson discovered the CMB [1] in 1964. They wanted to test a sensitive antenna which was intended for satellite communications [1]. The plan was to calibrate the antenna in an environment without radiation [1]. They adjusted the antenna to a wavelength of $\Lambda = 0.0735$ m [1]. They thought the chosen emission window should be quiet because it was between the longer wavelength emission from the Galaxy and the shorter wavelengths from the Earth's atmosphere [1]. The antenna was directed high above the galactic plane in order to avoid most of the scattered radiation from the Galaxy [1].

Penzias and Wilson detected a low level of background noise that radiated from every direction [1]. Since an intensity peak was not detected in the direction nearby the M31 galaxy in Andromeda, the noise was not likely from distant galaxies [1]. The noise neither varied with the altitude above the horizon nor as a function of the thickness of the atmosphere. Therefore, it could not originate in the earth's atmosphere [1]. No errors were found after retesting the antenna and electronics [1]. Penzias and Wilson concluded that the noise was excess radiation which uniformly filled the Universe [1]. According to their measurements, the radiation had a blackbody temperature of 3.5 K and was isotropic and unpolarized [1].

Robert Dicke and a group of physicists were theorizing about CMB radiation and preparing an attempt to measure it [1]. Once hearing of Penzias and Wilson's measurements, Dicke provided the theory behind the measurements [1]. They jointly published an article in 1965 about the cosmological implications [1]. In 1978, Penzias and Wilson were awarded the Nobel prize for the discovery of the CMB [1].

Further study on CMB has shown that it has a black body temperature of about 3 K (Kelvin). As scientists observe CMB, this temperature appears to be consistent in all parts of the universe. Thus CMB supports the idea that the universe is isotropic. From earth's point of view the radiation temperature is the same in every direction.

The CMB leads to a very significant conclusion which is a most precise determination of the total energy density parameter Ω_0 of the universe. Ω_0 is dimensionless and is represented by the equation

$$\Omega_0 = \frac{\rho_0}{\rho_c} = \frac{8\pi G\rho_0}{3H_0^2}\ . \qquad (20)$$

The present value of the density of the universe is represented by ρ_0 [1]. The critical density ρ_c defines the density at which the total energy of the universe is equal to zero and hence $k = 0$ [1]. G is the gravitational constant, $G = 6.673 * 10^{-11}$ m^3 Kg^{-1} s^{-2} and H_0 is the Hubble constant. Ω_0 is the sum of physical matter, radiation, and the cosmological constant or dark energy.

$$\Omega_0 = \Omega_m + \Omega_\Lambda. \tag{21}$$

The cosmological constant density parameter Ω_Λ is represented by the equation

$$\Omega_\Lambda = \frac{\Lambda}{8\pi G\rho_c} = \frac{\Lambda}{3H_0^2}. \tag{22}$$

Most conveniently, measurements of Ω_0 equal one. The universe appears to be in the privileged time where measurements of the total density parameter equal unity.

V. Supernovae Type Ia (SNeIa)

After Einstein provided the description of the cosmological constant (or what came to be known as dark energy), the next big step in the constant's development was the apparent discovery of universal acceleration through supernovae observations. The FLRW model could describe an expanding universe, but it could not describe an accelerating one without the introduction of a repulsive force. This repulsive force is most often attributed to dark energy or the cosmological constant. Finding that the universe is expanding is such an important event that I will include a description of the supernova stars that led to the finding. What follows is a description of the physical processes involved in supernovae Type One "a" (SNeIa) explosions (the specific type of supernovae which was used as a standard candle in order to detect acceleration).

A supernova (SN) is a star that explodes as it dies and becomes extremely luminous in the process. There are several different types of supernovae (SNe). The type of interest when studying dark energy is SNeIa. It consists of a binary star system composed of a heavy white dwarf and a red giant star.

White dwarfs have masses comparable to the sun but are of a size comparable to earth. Red giants are 10 or 100 times the radius of the sun and are the same or ten times the mass of the sun. Because the white dwarf has a large mass compressed in a small space, its gravitational field is much stronger than the red giant. The white dwarf accretes mass from the partner star.

Atomic elements inside the white dwarf fuse together to form larger elements. The dwarf is in an electron degenerate pressure state in which the increasing temperature from fusion does not affect the star. In the degenerate state, the electrons in the dwarf are so close that they cannot come any closer together. The force of gravity causing the star to collapse is balanced by the pressure of the degenerate particles not wanting to be closer together.

Once reaching the Chandrasekhar mass of 1.44 solar masses (~ 1.4 times the mass of the sun), the dwarf becomes unstable. Fusion has produced iron Fe in the core and, in a process called photodisintegration, the iron is converted into neutrons and neutrinos. The temperature is hot enough that the particles are no longer degenerate. Gravitational pressure quickly collapses the core. A bounce or shock wave occurs and enough explosive nuclear reactions occur that the dwarf explodes. The explosion completely destroys the star, but in the process causes the star to brighten by ~ 10^8 times its original brightness.

Of all the processes of the SNeIa that lead to the explosion, the most significant characteristic of these binary pairs is that each white dwarf explodes at the same Chandrasekhar mass. Therefore, each SNIa displays the same peak brightness, so the peak luminosity is intrinsic. For this reason, SNeIa are called standard candles. Any astronomical object whose intrinsic luminosity can be inferred independently of the observed brightness per area, or flux, is called a standard candle.

Mathematically, one can show how to find the distance of the SNeIa by using the luminosity distance relation

$$m - M = -5 + 5 \log d_L. \tag{23}$$

Here m is the apparent magnitude, M is the absolute magnitude, d_L is the distance in parsecs (pc) to the astronomical object. A parsec is $(3.096)(10^{16})$ meters. The apparent magnitude is a measure of the brightness of an astronomical object as an observer sees it from earth. The absolute magnitude measures the brightness of an astronomical object as if it was ten parsecs away. The brightness of an object increases as the value of m, M decreases. SNeIa have the same peak $M \sim -19.5$ [5]. Observers can measure m for SNeIa. Two of the three variables are known for the luminosity distance relation, so d_L can be calculated.

Two independent groups (High-Z Supernova Search Team [5] and Supernova Cosmology Project) began searching the universe for SNeIa which had a high redshift of $z \sim 1$. In 1998, the High-Z Supernova Search Team published an article concerning 16 high redshift SNeIa and compared them to 34 low redshift SNeIa. Scientists expected to find that the universe was decelerating. Observations from both supernova groups seemed to show otherwise. The High-Z Supernova Search Team and Supernova Cosmology Project concluded that the universe was accelerating. The observations also posed a problem for the FLRW model: the model would not agree unless the cosmological constant was added. The observations show that the theoretical d_L in an FLRW universe model without a cosmological constant was 10% to 15% too small [5]. With the cosmological constant added, the FLRW model is again able to theoretically support the observations of SNeIa and CMB. SNeIa most precisely determines $\Omega_0 = \Omega_\Lambda = \Omega_m$ as close to unity, which is in agreement with the CMB observations.

VI. Problems with Dark Energy

Since the FLRW model with dark energy is able to theoretically support the observational data of CMB and SNIa, one must wonder why some scientists have proposed other models. This section discusses several reasons.

One reason is that science should test many possible solutions to a problem. The first or most popular answer proposed should not be accepted without the study of other possibilities. The homogenous and isotropic FLRW model has been generally accepted, but there are other models as well. Another reason concerns the difficulty of observing homogeneity [6]. Observations of homogeneity are limited by position: scientists can only observe from the position of the earth/solar system.

The cosmological constant problem describes another reason why other universe models should be studied. In the FLRW model with dark energy, the energy density parameter of the cosmological constant Ω_Λ represents about 70% of the total energy density parameter of the universe Ω_0. The cosmological constant is usually interpreted as vacuum energy. Remember in quantum mechanics for a harmonic oscillator

$$E = (n + 1/2)\, h\omega. \tag{24}$$

Here E is energy, $n = 0$ is the ground state of a particle, $n = 1, 2, 3,...$ is an excited state, h is Plank's constant h over 2π, and ω is the angular frequency of the oscillator. Even at the ground state, there is energy

$$E = (1/2)\, h\omega. \tag{25}$$

Particle physics and quantum theory say that vacuum energy (ground state energy) is 10^{120} times the value obtained from the SNeIa data. In this respect, the FLRW model does not 100% theoretically support observations. The theory and experiment for dark energy disagree by a factor of 10^{120}. The cosmological constant problem also refers to the mysterious property of the cosmological constant fluid: negative pressure. The cosmological constant problem make dark energy unobservable except in cosmology.

Lastly, the coincidence problem points out a curious situation which arises in the FLRW model containing dark energy: at the present time the energy density parameter of matter Ω_m is nearly the same as that of dark energy Ω_Λ. If one adds Ω_m and Ω_Λ together, the total energy density parameter Ω_0 is close to unity. The coincidence problem asks why the present-day universe is in such a privileged position as when Ω_m and Ω_Λ are about the same, thus suggesting the Λ-dominated phase of the universe started in the "recent" past. In view of these problems with dark energy, some scientists have turned to studying an inhomogeneous universe and the Lemaître-Tolman-Bondi (LTB) model.

VII. Inhomogeneity and the Lemaître-Tolman-Bondi (LTB) Model

This section describes a second generally accepted type of universe model in addition to FLRW. This choice of models is characterized by the introduction of structural inhomogeneities. FLRW is the most commonly accepted model, while the Lemaître-Tolman-Bondi (LTB) model is the most popular of those based on an inhomogeneous and isotropic universe. The LTB model provides an explanation for the observations of CMB and SNeIa. It can be solved by evaluating the cosmological constant Λ in Einstein's equations, but is most frequently solved with Λ equal to zero. The observer in an LTB model is positioned in the center of a radial inhomogeneity in order to account for the isotropy of the CMB. Other inhomogeneous models place the observer in an anisotropic position to the side of the point of isotropy. In addition to describing the characteristics and development of the LTB model of the universe, various other inhomogeneous models are briefly mentioned.

The ideas for an inhomogeneous and isotropic universe date back to Georges Éduard Lemaître [7]. He published an article in 1933 after Alexander Friedmann theoretically predicted and Edwin Hubble had observed the first evidence of the expansion of the universe. Lemaître chose to take the cosmological constant as positive throughout his paper [7]. He described an inhomogeneous universe, *i.e.*, one that does not have the same density throughout. The structure and dynamics of a region of space depend on the position of the observer. For example, peanut butter would be inhomogeneous if someone tossed chocolate chips into the jar and stirred it. The chips would not be uniformly mixed into the peanut butter. In the same way, the universe could have pockets of high matter density and also voids where there is no matter.

In 1934, Richard C. Tolman considered several different universe models [8]. He considered inhomogeneous models of low density dust (nebulae). Tolman solves Einstein's field equations in reference [8]. He goes on to describe the Static Einstein Model, Distorted Einstein Model, Non-Static Friedmann Model, Distorted Friedmann Model, and Combination of Uniform Distributions [8]. Tolman cautions that homogeneous models should not be accepted as the only option [8]. He warns about making extrapolations that are too large using homogeneous models since inhomogeneous models theoretically fit as well.

M. Hossein Partovi and Bahram Mashhoon concluded in 1984 that local models with radial inhomogeneities are indistinguishable from the standard homogeneous models [9]. If inhomogeneity, anisotropy, and shear are not considered, the only solutions to Einstein's equations consistent with observations are the FLRW model [9]. Shear is present in an expanding universe when the radial and lateral velocity of matter differ. Due to shear, a spherical object can become a spheroidal. Partovi and Mashhoon proposed to study the parts of the universe which can be repeated by a homogeneous model. They also tested observations by using an inhomogeneous but isotropic model. As stated above, they found the observations can be explained through either homogeneity or inhomogeneity. Partovi and Mashhoon's general but local universe model includes the specific one considered by Lemaître [7] and Tolman [8]. In 1984, Bahram Mashhoon solved Einstein's equations using a universe model with radial inhomogeneities due to shear [6].

In 1992, Hannu Kurki-Suonio and Edison Liang used redshift surveys to map the distribution of matter (in the local universe) in the spherically symmetric Lemaître-Tolman-Bondi (LTB) model with concentric shells of dust (inhomogeneities) [10]. Combining the distance equation with constraints from solutions to Einstein's equations the LTB model yields

$$ds^2 = -dt^2 + \frac{(\frac{\partial R}{\partial r})^2}{1 + 2E(r)} + R^2 d\Omega^2 \tag{26}$$

where R is a scale factor that is dependent on time t and position r, and $2E(r)$ is related to spatial curvature. Kurki-Suonio and Liang determined that it is not self-consistent to use the homogeneous-isotropic FLRW model without including dark energy to explain a periodic relation of the distance between galaxies and their recessional velocities. In other words, the FLRW model could not use inhomogeneities to explain the relationship between the distance and velocity of galaxies (Edwin Hubble discovered this relationship as mentioned earlier in this paper). Kurki-Suonio and Liang used the inhomogeneous LTB model to explain the distance-velocity relationship. They explained that the LTB model could explain the observation of temporary homogeneity due to the interaction of inhomogeneities. They discuss two types of inhomogeneity: one due to space and the other to time. They concluded that temporary homogeneity could be observed due to the interaction of spatial and time-dependent inhomogeneity.

M. A. H. MacCallum provides a review in 1992 of inhomogeneous and anisotropic models of the universe [11]. He covers only exact equations (as opposed to using perturbation theory) concerning Einstein's equations [11]. MacCallum organizes the models in the review according to mathematical classifications. In his opinion, the pure dust LTB universe is only good for modeling; and he does not think the present universe is structured in accordance with the LTB model [11].

After MacCallum reviewed the different inhomogeneous models of the universe, Humphreys, Maartens, and Matravers calculated in 1996 the observational relations such as CMB in the LTB universe models [12]. Observations show that the CMB has a dipole which causes its measurement in one direction to be different from that in another. Scientists studying the FLRW universe normally attribute the dipole to the motion of the earth with respect to the rest frame of the CMB. This motion is called peculiar velocity. Using the LTB model, Humphreys, Maartens, and Matravers explore another possible situation. The observer is off-center from the LTB universe and is comoving so that the mean matter flow appears isotropic. They are able to reproduce the observed CMB dipole and obtain exact expressions for the Hubble constant H and deceleration parameter q(measurement of the deceleration or acceleration of the expanding universe) that both have quadrupole anisotropies [12].

In 1998, Andrzej Krasiński discussed how inhomogeneity in the LTB model can generate anisotropic CMB radiation [13]. The inhomogeneity is assumed to be in the vicinity of the center of symmetry at $r = 0$ [13]. The model tends toward the FLRW model as r approaches infinity. Light-rays emitted by scattering must pass though the inhomogeneity along different distances from $r = 0$ [13]. Integration of the null geodesic equations in the LTB model gives the influence of inhomogeneity on the black body temperature of the CMB radiation. The temperature is found from the equation

$$\frac{T_{obs}}{T_{em}} = \frac{1}{1+z} = \frac{(k^{\mu}u_{\mu})_{obs}}{(k^{\mu}u_{\mu})_{em}} \tag{26}$$

where "*obs*" refers to the observation event, "*em*" refers to the emission event, k^{μ} is the calculated tangent vector to the null geodesic (the tangent line to curved spacetime has zero norm or length) which should be close to zero, and u_{μ} is the velocity vector of the dust medium, and z measures the redshift of light due to the Doppler effect [13]. T_{em} is the same for all rays [13].

In the 1990's, extensive research was performed in order to study SNeIa. The magnitude-redshift relation of the SNe showed that the universe was expanding at an accelerated rate.

Ordinary matter (that which consists of neutrons, electrons, and protons) decelerates the expansion of the universe due to mutual gravitational attraction. Scientists predominantly ascribe the surprising acceleration of the universe to a repulsive force which counteracts the attractive gravity of ordinary matter. The repulsive force was coined as "dark energy" by Michael Turner in 1998. Scientists have proposed different sources of a repulsive force. Although there is more than one possible source of dark energy, this paper focuses on dark energy cause by the cosmological constant which Einstein began to describe in the early twentieth century.

The term "dark" is appropriate because dark energy has properties different from all other accepted concepts in physics. Due to such peculiarities, Marie-Noëlle Célérier and other scientists looked to other explanations (such as the LTB model) before accepting dark energy as the explanation of the accelerating universe.

In 2000, Marie-Noëlle Célérier published her article “Do We Really See a Cosmological Constant in the Supernovae Data?” [14]. Célérier's answer differs from the majority of scientists who use the cosmological constant or dark energy in their theories and observational interpretations of the universe [14]. If the homogeneous, isotropic FLRW model of the universe with dark energy included can explain observations such as the apparent acceleration of the universe, what would cause Célérier to look for alternatives to dark energy? She mentions the peculiar discovery that the cosmological constant (dark energy) is "many orders of magnitude smaller than the energy of the vacuum expected in standard particle physics models" [14]. She turns to the "less exotic" explanation of Georges Lemaître and the inhomogeneous Lemaître-Tolman-Bondi universe model [14].

Célérier uses the inhomogeneous LTB model to explain the theories and observations previously used to describe a homogeneous universe. She uses the magnitude-redshift relations from the SNe and solves Einstein's equations. These solutions show that inhomogeneities can account for acceleration just as a cosmological constant.

In 1999, J. F. Pascual-Sánchez showed the cosmological constant can be taken as zero [15]. He used a model different from LTB. His model yields isotropic CMB.

Since 2000, many scientists continued to study the effects of inhomogeneities [3, 16 - 21]. Their calculations show that inhomogeneities can account for the observations of SNIa and CMB. They conclude that dark energy is not the only possible explanation of the observations. Garfinkle notes that neither FLRW nor LTB model perfectly represents the universe [20]. Both models are over-simplified when compared to the complexity of the universe.

In 2003, the authors of reference [22] studied local inhomogeneities in an FLRW model with a positive cosmological constant; they successfully explain CMB and SNIa data. The authors of reference [23] also study spatially inhomogeneous cosmological models with a positive cosmological constant. Scientists made further studies on CMB and wrote the data in [24, 25]. The Wilkinson Microwave Anisotropy Probe (WMAP) showed that CMB was slightly anisotropic. This observation does not disprove the isotropic 3 K temperature of CMB. The anisotropies are primarily due to the movement of the earth which creates a Doppler effect in CMB observations as well as other subtle effects.

In 2005, George F. R. Ellis and Thomas Buchert discussed small-scale inhomogeneities in a large-scale homogeneous universe model [26]. They tested whether the small-scale inhomogeneities had a possibility of affecting observations. J. W. Moffat studied the LTB model [27]; he concluded that the presence of inhomogeneities could cause a change in observational and measurable data: Hubble constant H_0, acceleration parameter q_0, and CMB [27].

In 2006 Marie-Noëlle Célérier wrote a review on the structure of the universe [28]. She discusses the FLRW, LTB, and other inhomogeneous models. Syksy Räsänen also wrote a review in 2006 concerning universal "acceleration"due to inhomogeneity and anisotropy [29]. The LTB model is described as the "Onion" model by [30]. The observer can be in the center or off-center of the "Onion." By solving Einstein's equations from either position, the authors of [30] conclude that large scale fluctuation (inhomogeneity) can mimic dark energy. If the repulsive force (dark energy) is a mimic, then the universe would decelerate as oppose to accelerate.

In 2006, J. W. Moffat described an accelerating universe in which the model is inhomogeneous for redshifts of $z > 20$ and homogeneous for $z < 20$ [31]. The deceleration parameter q is set so that the cosmological constant (dark energy) is not needed. The authors of [32] describe the characteristics of a universe represented by the LTB model. Reza Mansouri studied an LTB universe model which is embedded in a background FLRW model [33]. The observer can be positioned anywhere in the universe, and this model is able to solve for the energy density parameter Ω and the deceleration parameter q. In 2007, Antonio Enea Romano uses the LTB model to mimic the "acceleration" of the universe [21].

As seen from the research above, both dark energy and inhomogeneity provide solutions to Einstein's equations and explain CMB and SNIa observations. One must now compare the advantages and disadvantages in order to decide which solution best describes the universe.

VIII. Advantages and Disadvantages of Inhomogeneity

Inhomogeneity is a more natural solution to "acceleration" compared to dark energy, but inhomogeneity is more mathematically complicated.

The first advantage is that local inhomogeneity is easily observable. No special equipment is needed. For example, just within the room there are different densities of furniture, pictures, or decorations. The earth and solar system are filled with local inhomogeneities. A large-scale inhomogeneous universe is reasonable. Second, there are no conceptually difficult or rare characteristics describing inhomogeneities as those describing dark energy. Inhomogeneity does not suffer from the coincidence problem. As mentioned previously, a large-scale inhomogeneous universe is logical since it exhibits local inhomogeneities.

Despite the advantages of inhomogeneities, dark energy is still more frequently accepted. One reason for this is because solving Einstein's equations with inhomogeneities is cumbersome.

The FLRW universe is the simplest model. Models including inhomogeneities (such as the LTB model) are only able to match observations from part of the universe. Multiple inhomogeneous models are needed to describe the total universe. Both dark energy and inhomogeneities have advantages and disadvantages.

IX. Possible Future Studies and Conclusion

Scientists do not know whether the universe is characterized by dark energy or inhomogeneity. One possible future study is to continue the research for SNIa at high redshifts. This includes increasing the number observed, improving techniques for observation, and observing SNIa at higher redshifts. Another future study is to search for other astronomical objects which can be used as standard candles.

Mathematically both dark energy and inhomogeneities provide solutions to Einstein's gravitation equations. Both are able to explain observations such as CMB and SNIa. A homogeneous isotropic universe with a positive cosmological constant is equally plausible as an inhomogeneous isotropic universe with a cosmological constant equal to zero. In conclusion, dark energy and inhomogeneity can equally be used to describe the structure of the universe.

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